

AN UNIVERSAL MODEL FOR LOSSY AND DISPERSIVE TRANSMISSION LINES  
FOR TIME DOMAIN CAD OF CIRCUITS

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**ABSTRACT**

An universal equivalent circuit for lossy and dispersive transmission lines is presented. Existing CAD packages, such as SPICE, can be used for its implementation. The starting point for obtaining the model are the analog filters which approximate the forward impulse response and characteristic impedance. The equivalent circuit is used to simulate the effects for pulse propagation on microstrip transmission lines. An examination of the validity of the model is carried out analyzing the response for an example case in the time and frequency domains.

**INTRODUCTION**

As circuit speeds increase, more attention must be paid to the design of interconnects. Improperly designed interconnects can result in increases of signal delay because of losses, inadvertent switching and noise as a result of crosstalk [1]. With subnanosecond rise times, the length of interconnects can become a significant fraction of a wavelength. Consequently, the conventional microstrip transmission line models, which generally assume ideal lines, are not adequate in this case. In [2] a model, compatible with SPICE, for lossy and dispersive transmission lines has been proposed. This model, however, requires a great quantity of lumped elements sections and therefore is not useful in most practical cases.

In this paper a technique which allows to obtain an universal equivalent circuit in order to model this class of structures is presented.

**TRANSMISSION-LINE MODELLING TECHNIQUE**

**Basic principles**

Lossy and dispersive transmission lines have parameters  $-Z_0(s), \gamma(s)$  - dependent on frequency. In order to develop a general circuit model for time domain CAD, it is necessary to simulate these effects efficiently. This involves synthesizing a symmetrical network with both, a frequency dependent characteristic impedance and a lossy and nonlinear phase propagation function. Standard circuit design synthesis methods foreither frequency dependent one-ports (impedances) or

two-ports with any propagation function but with a constant characteristic impedance exist [3]. As it will be shown, in the proposed technique the general problem is broke into these two using an adequate network that includes linear voltage dependent voltage sources and the so designed impedances and two-ports. An efficient way of designing those impedances and two-ports with few lumped elements and ideal transmission lines is also developed.

**Implementation of the transmission line model**

**a) Generalized method of characteristics**

Transmission lines can be characterized by the following equations:

$$V_1(s) = Z_0(s)I_1(s) + F(s)[V_2(s) + Z_0(s)I_2(s)] \quad (1)$$

$$V_2(s) = Z_0(s)I_2(s) + F(s)[V_1(s) + Z_0(s)I_1(s)] \quad (2)$$

where  $F(s) = \exp[-\gamma(s)l]$  is the propagation function and  $(V_1(s), I_1(s))$  and  $(V_2(s), I_2(s))$  are the terminal voltages and currents at the near and far end of the transmission line, as shown in fig.1.

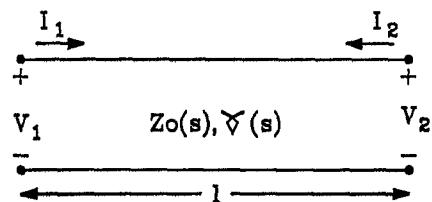


Fig. 1. Transmission line relationships

The previous equations suggest using a transmission line model as shown in fig.2.

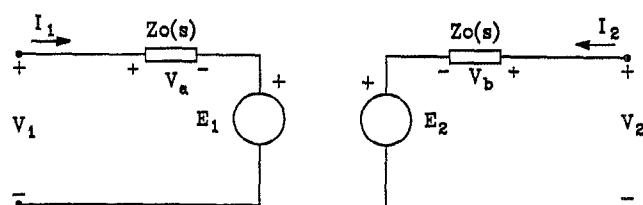


Fig. 2. The characteristic model of a transmission line

where:

$$V_a(s) = Z_o(s) \cdot I_1(s) \quad (3)$$

$$V_b(s) = Z_o(s) \cdot I_2(s) \quad (4)$$

$$E_1(s) = F(s) \cdot V_p(s) = F(s)[V_2(s) + V_b(s)] \quad (5)$$

$$E_2(s) = F(s) \cdot V_q(s) = F(s)[V_1(s) + V_a(s)] \quad (6)$$

Networks that implement the impedance  $-Z_o(s)$ - and the voltage sources  $-E_1(s), E_2(s)$ - must be found.

The first is an impedance synthesis problem. The voltage  $E_1(s)$  -or  $E_2(s)$ - can be obtained from the propagation of the voltage  $2V_p(s)$  -or  $2V_q(s)$ - across a two-port as long as its propagation function is  $F(s)$  and its arbitrary characteristic impedance,  $R_n$ , is the same as the circuit loads, as shown in fig.3.

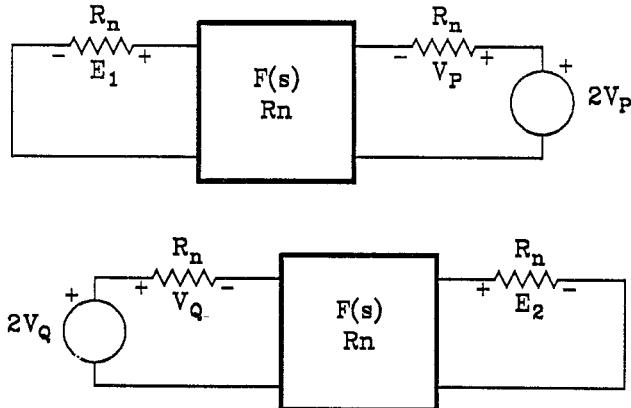


Fig.3. Circuits to obtain the voltages  $E_1$  and  $E_2$

If in the circuit of fig.2 we replace the sources  $E_1$  and  $E_2$  by linear unity gain voltage controlled voltage sources depending upon the  $E_1$  and  $E_2$  voltages of fig.3 we will have solved our problem as the impedance and the two-port can be designed with standard methods. However, as circuits 3a and 3b are the same except for the excitation, the superposition principle can be used to obtain the results shown in fig.4.

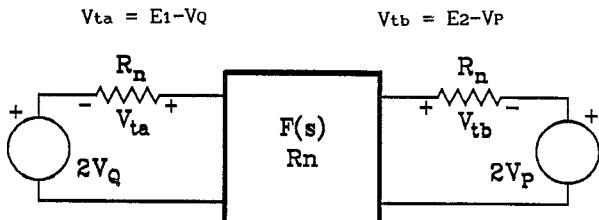


Fig.4. Application of superposition principle on circuit of previous figure

Now  $V_{ta}$  and  $V_{tb}$  are obtained instead of  $E_1$  and  $E_2$ , and therefore, additional voltage sources must be placed in the circuit of fig.2 adding  $V_p$  and  $V_q$  to  $V_{ta}$  and  $V_{tb}$ , giving finally the circuit of fig.5.

### b) Synthesis of the Characteristic Impedance and the Propagation Function

In accordance with the method described in [4], the analog filters used for  $F(s) = \exp[-\gamma(s)t]$  and  $Z_o(s)$  that take account of dispersive effects of the modeled transmission line, are:

$$F(s) = k_1 \prod_{j=1}^{N_1} \frac{(s+c_j)}{(s+d_j)} \prod_{i=1}^{N_2} \frac{(s^2-f_{1i}s+f_{0i})}{(s^2+f_{1i}s+f_{0i})} \exp(-st) \quad (7)$$

and

$$Z_o(s) = k_2 \prod_{j=1}^{N_2} \frac{s^2+g_js+h_j}{s^2+e_js+b_j} \quad (8)$$

The term  $\exp(-st)$  takes into account a constant delay, implemented with an ideal transmission line, so that the rest of the frequency dependent propagation delay is easier to obtain with simpler networks.

The coefficients of the previous expressions are found by a least-squares matching procedure from available data in the frequency domain for the effective dielectric constant [5] and characteristic impedance, or starting from values of  $R$ ,  $L$   $G$  and  $C$  of the line [6]. Stability and realizability considerations must be taken into account on the minimisation procedure.

The lumped elements of the model are obtained by synthesis of functions (7) and (8) by conventional techniques.

### COMPUTATIONS

In order to validate the accuracy of the modeling technique and demonstrate the usefulness of the model, the distortion of a 45 psg wide -at the half magnitude points- gaussian pulse travelling along a 122.142mm long microstrip line is computed. The line substrate is alumina ( $\epsilon_r=10.2$ ), and the line parameters are:  $H=0.635\text{mm}$ ,  $t=0.0017\text{mm}$ ,  $R_s=8.63 \times 10^{-3}\sqrt{\text{f}}$  and  $\tan\delta=0.0015$ .

Figure 6 shows good agreement between the propagation characteristics of the obtained circuit and values computed for the line. Figure 7 shows the SPICE program together with the equivalent circuit and values of the elements. Finally the results of the analysis performed with the SPICE and using DFT techniques are compared in fig.8, showing good agreement.

### CONCLUSIONS

A technique to develop an efficient and universal model for lossy and dispersive transmission lines has been presented. The developed model is compatible with time domain commercial programs, such as SPICE.

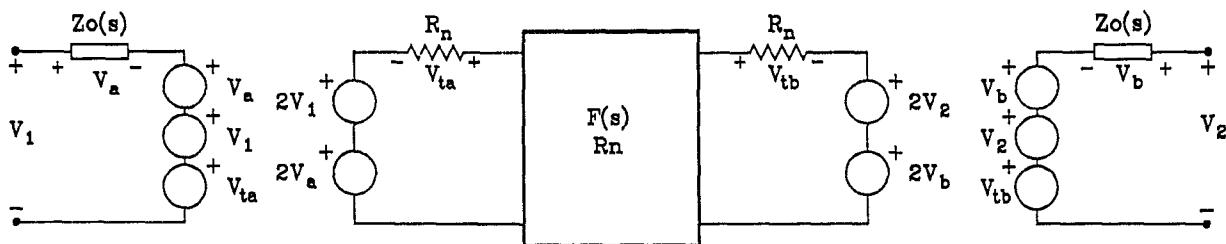


Fig. 5. Universal transmission line circuit model

#### ACKNOWLEDGEMENTS

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- [2] V.K. Tripathi and A. Hill, "Equivalent Circuit Modeling of Losses and Dispersion in Single and Coupled Lines for Microwave and Millimeter-wave Integrated Circuits", *IEEE Trans. Microwave Technique*, Vol. 36, No. 2, pp. 256-262, Feb. 1988.
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- [6] R.E. Collin, "Foundations for Microwave Engineering", McGraw-Hill, 1966, pp. 87-89.

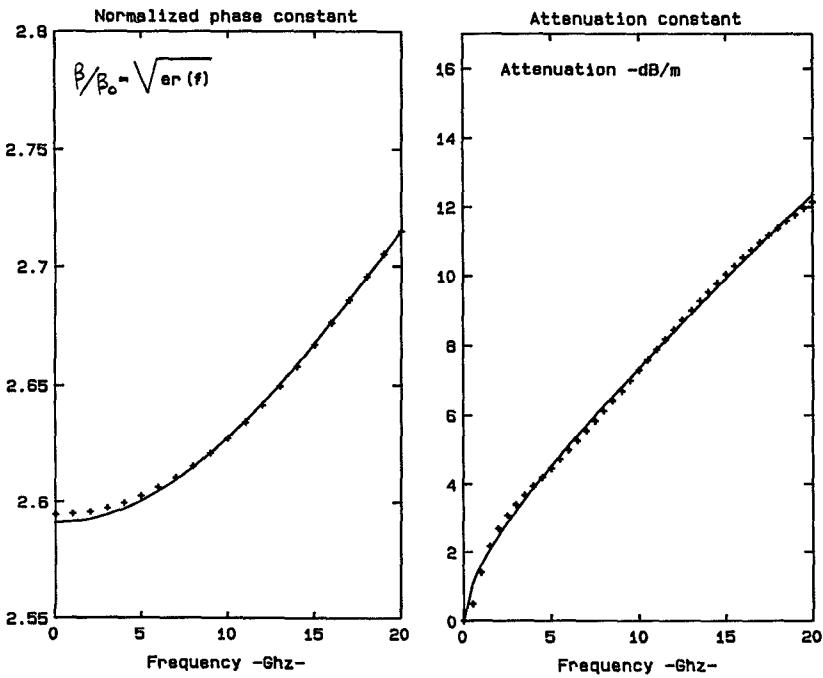


Fig. 6. Propagation characteristics of the equivalent circuit model. (—) Computed for the line; (+ + +) Values calculated for the model

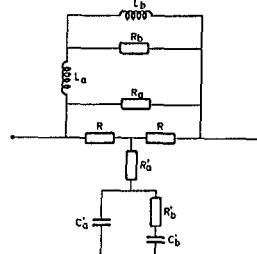


figure 7.a. Network synthesis of the attenuation function

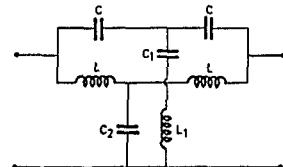


figure 7.b. Network synthesis of the propagation function

R	$\Omega$	53.7
R_a	$\Omega$	4.76728655
R_b	$\Omega$	0.87580327
L_a	pH	53.1985759
L_b	pH	93.0902279
R_a'	$\Omega$	604.903954
R_b'	$\Omega$	3292.62299
C_a'	pF	0.01844809
C_b'	pF	0.03228163

L	nH	0.35007474
C	pF	0.09221530
L_1	nH	0.55310334
C_1	pF	0.08538790
C_2	pF	0.18443070

values of the elements

TABLE  
SPICE INPUT DATA FILE FOR THE DEVELOP MODEL

\*\*\* GAUSSIAN PULSE \*\*\*

```
VIN1 5 0 PWL( 410ps .0000 415ps .0001 420ps .0002 425ps .0005 430ps .0012
+ 435ps .0031 440ps .0072 445ps .0159 450ps .0326 455ps .0625 460ps .1118
+ 465ps .1869 470ps .2916 475ps .4250 480ps .5783 485ps .7349 490ps .8720
+ 495ps .9663 500ps 1.000 505ps .9663 510ps .8720 515ps .7349 520ps .5783
+ 525ps .4250 530ps .2916 535ps .1869 540ps .1118 545ps .0625 550ps .0326
+ 555ps .0159 560ps .0072 565ps .0031 570ps .0012 575ps .0005 580ps .0002
+ 585ps .0001 590ps .0000 )
```

\*\*\* UNIVERSAL EQUIVALENT CIRCUIT MODEL \*\*\*

```
RG 5 1 53.7
```

```
RO1 1 2 53.7
```

```
RO2 7 9 53.7
```

```
RO3 10 13 53.7
```

```
RO4 12 17 53.7
```

```
EA 3 2 2 1 1
```

```
E1 4 3 0 1 1
```

```
ETB 0 4 7 9 1
```

```
E1I 7 6 1 0 2
```

```
EAI 6 0 1 2 2
```

```
EB 15 12 12 17 1
```

```
E2 16 15 0 17 1
```

```
ETA 16 0 10 13 1
```

```
E2O 13 14 17 0 2
```

```
EBO 14 0 17 12 2
```

```
T1 9 0 25 0 Z0=53.7 TD=858.099165ps
```

```
X1 25 0 10 SIMUL
```

```
RL 17 0 53.7
```

```
.SUBCKT SIMUL 1 2 3
```

```
XX1 1 2 4 CIRC
```

```
XX2 4 2 5 CIRC
```

```
XX3 5 2 3 CIRC
```

```
.ENDS SIMUL
```

```
.SUBCKT CIRC 1 2 8
```

```
XXX1 1 2 3 ATEN
```

```
XXX2 3 2 4 PTI
```

```
XXX3 4 2 5 PTI
```

```
XXX4 5 2 6 PTI
```

```
XXX5 6 2 7 PTI
```

```
XXX6 7 2 8 PTI
```

```
.ENDS CIRC
```

```
.SUBCKT ATEN 1 2 3
```

```
R1 1 4 53.70
```

```
R2 4 3 53.70
```

```
RA1 1 3 4.767286
```

```
LA1 1 5 53.1985759pH
```

```
RA2 5 3 .8758032
```

```
LA2 5 3 93.0902279pH
```

```
RA3 4 6 604.903954
```

```
RA4 6 7 3.29262297K
```

```
CA1 6 2 .018448092pF
```

```
CA2 7 2 .032281530pF
```

```
.ENDS ATEN
```

```
.SUBCKT PTI 1 2 3
```

```
CPT1 1 4 0.0922154pF
```

```
CPT2 4 3 0.0922153pF
```

```
CPT3 4 5 0.0853879pF
```

```
RPT3 4 5 1000MEG
```

```
CPT4 6 2 0.1844307pF
```

```
LPT1 1 6 .35007474nH
```

```
LPT2 6 3 .35007474nH
```

```
LPT3 5 2 .55310334nH
```

```
.ENDS PTI
```

```
.PROBE V(17)
```

```
.TRAN 5.000p 2.000n 1.300n 0
```

```
.PRINT TRAN V(17)
```

```
.END
```

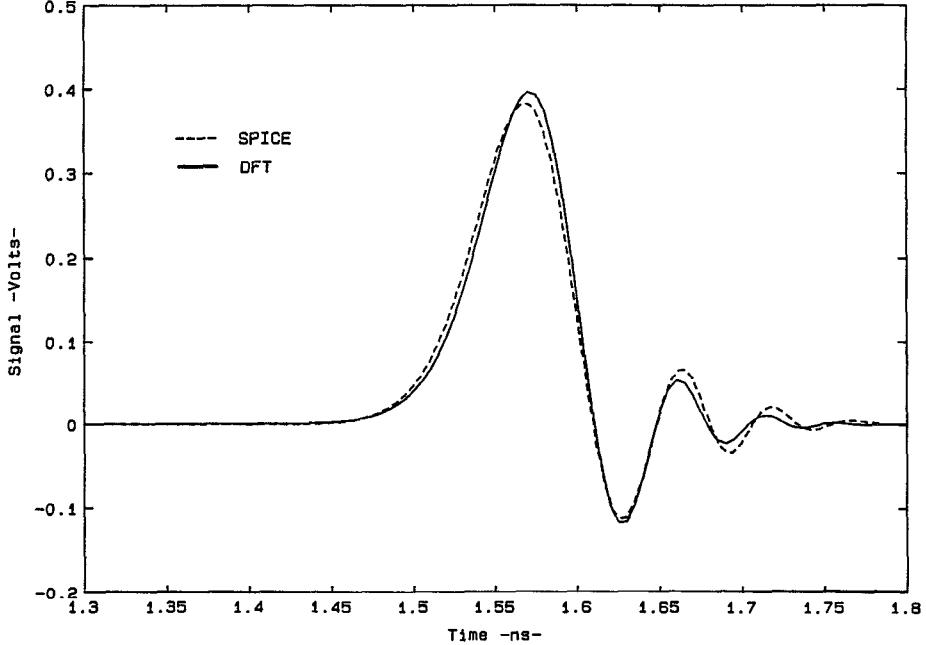


Fig. 8. Gaussian DC pulse dispersion at a distance  $L=122,142\text{mm}$ . (---) Response of the model developed with SPICE; (—) Computed by using DFT techniques